For a given $n$, a sum tree for $n$ is a full binary tree whose root is labeled $n$, whose leaves are labeled 1, and in which the children of a node labeled $j$ have labels that sum to $j$. Every node should have a label $> 0$.

Given a cost table $c[i]$, the cost of a node labeled $i$ is $c[i]$, and the cost of the tree is the sum of the cost of its nodes.

a. What is the cost of a sum tree for 1? (in terms of $c[1]$)

b. What is the cost of a sum tree for 2? (in terms of $c[1]$ and $c[2]$)

c. An optimal sum tree for $i$ is a sum tree for $i$ of minimum cost. What is an optimal sum tree for 8 if $c[1]$ to $c[7]$ are 1, 3, 12, 20, 20, 22, 25, 26?

d. Write down a formula that expresses the cost of an optimal sum tree in terms of the cost of smaller optimal sum trees. (Hint: In an optimal sum tree, the trees rooted at the left and right children of the root form optimal sum trees.)

e. Use the dynamic programming technique to design an algorithm that finds the cost of the optimal sum tree for $n$, given $n$ and $c[i]$, for $1 \leq i \leq n$.

f. Analyse the time that your algorithm takes (asymptotic notation).