Robust megavoltage x-ray spectra estimation from transmission measurements

Marian Manciu\textsuperscript{a*}, Felicia S. Manciu\textsuperscript{a}, Teodor Vulcan\textsuperscript{a}, Elena Nes\textsuperscript{b} and Robert G. Waggener\textsuperscript{b}

\textsuperscript{a}Physics Department, University of Texas at El Paso, TX, USA
\textsuperscript{b}Radiation Oncology Division, Radiology Department, University of Texas Health Science Center at San Antonio, TX, USA

Received 29 October 2008
Revised 27 February 2009
Accepted 6 March 2009

Abstract. Megavoltage X-ray sources are commonly used for therapy planning, and knowledge of their spectral distribution is important for accurate dose calculations. There are many methods that could provide reasonable estimations of Megavoltage X-ray spectra, when very accurate attenuation data or at least very good set of initial guesses of the spectra are available. We present here a novel method, which can be used for accurate Megavoltage spectral reconstruction without any prior knowledge of spectral distribution; the method performs well even when the available transmission data are affected by noise. The method is based on a search for a smooth function that minimizes the differences between measured and calculated attenuation data. The algorithm is compared with well-known existing algorithms, using computer simulated data, both error-free and containing added random Gaussian noise. The reconstructed spectra are subsequently used to calculate the transmission through 50 cm of bone, muscle or fat tissue. It is shown that the relative errors in dose calculations, using the spectra reconstructed via this method, are significantly smaller than those obtained via well-established reconstruction algorithms – Truncated Singular Value Decomposition (TSVD) and Expectation Maximization (EM).

These results suggest that the novel algorithm might be practical for routine Megavoltage therapy X-ray source calibration.

Keywords: Photon spectra, megavoltage photon beams, spectral reconstruction, bremsstrahlung, attenuation curves, inverse problem, linear accelerators

1. Introduction

In therapy planning, knowledge of the X-ray source spectrum is important for an accurate determination of the radiation dose absorbed by the patient. Long ago, Silberstein [1] suggested the use of attenuation measurements for the spectral reconstruction of a narrow X-ray beam. Since the attenuation coefficients of X-rays depend on their energy, the transmission at various depths is related to the spectrum via an integral equation, which (at least in principle) can be solved exactly. Unfortunately, the resulting system of equations is ill-conditioned, and the inherent experimental errors of the attenuation measurements

\textsuperscript{*}Corresponding author. E-mail: mmanciu@utep.edu.
make an accurate direct spectral reconstruction impossible. To circumvent these difficulties, many numerical techniques have been devised since the pioneering Silberstein work. These methods can be roughly classified into two categories: (1) parametric models, with only a few parameters to be optimized to reproduce the experimental attenuation data and (2) numerical inversion of the system of linear equations. The methods from the first category [2–6] avoid the solving of the ill-conditioned system, and it was shown that they are suitable for Megavoltage spectral estimation, when attenuation data with low uncertainties are available [5]. For maximum photon energies between 4 and 30 MeV and a broad range of filtration conditions, it was shown that the estimated data reproduced the input data within 0.1% in all cases, which was of the same order of magnitude as the uncertainties involved in its calculations [5]. When a forth parameter was involved, the maximum photon energies have been recovered with an accuracy of about 3% [5]. These results have been supported by experiment [6]. It is, however, expected that spectrum cannot be reproduced as well by only three of four parameters if many filters of unknown composition and thickness would be used (e.g., the spectrum of the beam exiting a patient). The advantage of the second category of the methods is that, at least in principle, no initial assumptions should be made about the spectrum. From this category, we note direct inversion [1,7], the use of the Moore-Penrose pseudoinverse, [8] or least squares methods [9].

Two novel techniques have emerged recently for numerical inversion of the system of linear equations. An iterative method has been proposed by Waggener et al. [10], in which an initial guess of the spectrum is selected and its corresponding attenuation at various depths is calculated. Then, the spectrum is successively adjusted until the calculated attenuations matches the measured attenuation data. Another method proposed recently by Sidky et al. [11] is based on the Expectation-Maximization algorithm, in which successive spectra are generated in order to minimize the Kullback-Liebler distance to the attenuation data. Both methods are extremely successful, if a very good initial guess of the spectrum is known and if accurate transmission data are available. Unfortunately, these requirements are often not met in clinical practice. The purpose of this article is to present a novel method for spectral reconstruction, which employs a search for a smooth function that minimizes the differences between the calculated and measured transmission data. The method is tested on computer simulated data for transmission through successive layers of water, both error-free and with added random Gaussian noise. It is shown that a reasonable estimation of the spectrum can be obtained, even when a high noise level affects the transmission data, without using any initial “guess” spectrum. The only a priori knowledge of the bremsstrahlung spectrum is the assumption that is a non-negative smooth function. This method for spectral estimation is tested on the transmission data obtained from Megavoltage beams for various acceleration potentials [12]. It is, clearly, expected that any method that tries to adjust the spectral distribution to minimize the differences between the input and calculated data will reproduce the former accurately. Therefore, in order to estimate the errors in dosimetry calculation due to a lack of (or insufficient) knowledge of the spectral distribution, spectra estimated using various methods (based on the Expectation-Maximization (EM), Truncated Singular Value Decomposition [13] (TSVD)) and the algorithm based on Total Variation Regularization (TVR) proposed here, have been employed to calculate transmissions through 50 cm of bone, muscle and fat tissues. It is shown that for the TVR reconstruction, the relative error at any depth (compared with the transmission of the true megavoltage spectrum) is less than 0.1%, which makes the algorithm practical for routine calibration of Megavoltage X-ray sources. The TVR algorithm has been tested for all spectra calculated via Monte-Carlo, for various types of linear accelerators and acceleration potentials, provided by Sheikh-Bagheri and Rogers [12].
2. Mathematical method

2.1. The inverse problem

The transmission $t$ through a slab of thickness $d$ of a polyenergetic X-ray beam is given by the linear integral equation:

$$t(d) = \frac{1}{\Phi} \int_{E_{\text{min}}}^{E_{\text{max}}} \phi(E) \exp(-\mu(E)d) \, dE$$  \quad (1)

where $\phi(E)$ represents the spectral distribution and

$$\Phi = \int_{E_{\text{min}}}^{E_{\text{max}}} \phi(E) \, dE$$  \quad (2)

is the fluence of the beam. Equation (1) (which neglects non-linear effects) would formally relate $t(d)$ and $\phi(E)$ via a direct Laplace Transform if $\mu(E)$ were proportional to $E$, whereas it would provide no information for the spectral distribution $\phi(E)$ if $\mu$ were independent of $E$. In practice, $\mu(E)$ is a slowly varying function of $E$, which makes the solution of the integral Eq. (1) an ill-conditioned problem. A typical solving method is to transform the linear integral equation into a system of linear equations by splitting the energy interval into $N$ bins of width $\Delta E = \frac{E_{\text{max}} - E_{\text{min}}}{N}$ and calculating the transmission values for $M$ thicknesses $d_m$: [10,11]:

$$t_m \equiv t(d_m) = \frac{1}{\Phi} \sum_{n=1}^{N} A_{mn} c_n$$  \quad (3)

where the intervals $\Delta E$ are chosen small enough such that the approximation:

$$A_{mn} c_n = \int_{E_n - \Delta E/2}^{E_n + \Delta E/2} \phi(E) \exp(-\mu(E)d_m) \, dE \approx \exp(-\mu(E_n)d_m) \int_{E_n - \Delta E/2}^{E_n + \Delta E/2} \phi(E) \, dE$$  \quad (4)

holds, with:

$$A_{mn} = \exp(-\mu(E_n)d_m)$$  \quad (5a)

and

$$c_n = \int_{E_n - \Delta E/2}^{E_n + \Delta E/2} \phi(E) \, dE$$  \quad (5b)

Correspondingly, the normalization relation (2) becomes

$$\Phi = \sum_{n=1}^{N} c_n$$  \quad (6)
The system of Eq. (3), with \( m \) running from 1 to \( M \) and \( n \) running from 1 to \( N \) can be formally reversed only for \( N = M \). In this case,

\[
c_n = \sum_{m=1}^{M} (A_{nm})^{-1} t_m
\]

(7)

In practice, because of the weak dependence of \( \mu \) on energy, the determinant of the matrix \( (A_{mn}) \) is so small that the numerical inversion of the matrix is problematic. Large values for \( M \) increase this numerical instability, while small values for \( N \) imply that the spectrum is described very roughly and inaccurately. The values typically used in the literature \([10,11]\) are of the order of \( N = 100 \) energy bins and \( M = 20 \) transmission measurements, which lead to an underdetermined system (Eq. (3)). While a formal solution is in this case impossible, the Moore-Penrose generalized inverse, based on Singular Value Decomposition (SVD) \([13]\) provides a very good spectral estimation of the \( N = 100 \) coefficients \( c_n \) using only \( M = 20 \) equations, when the values of the attenuation data \( t_m \) are extremely accurate, as will be shown later. However, it will also be shown in what follows that, when even small levels of random Gaussian noise are added to the computer simulated attenuation data, the spectrum reconstructed with the Moore-Penrose pseudoinverse fails to approximate the real one accurately.

To investigate the accuracy of the new algorithm and to compare it with the existing methods, we will use computer simulated data, either error-free or with added random Gaussian noise. We will evaluate the reconstruction algorithm for a realistic spectral distribution, calculated using the BEAM Monte-Carlo code by Sheikh-Bagheri and Rogers \([12]\), for a Varian linear accelerator and a maximum acceleration potential of 18 MV. Also, the method will be tested for the other spectral distributions calculated in \([12]\).

Since the attenuation measurements involve, in general, a large number of photons, in what follows we will assume that the transmission data are affected by random Gaussian noise, which is proportional to the square root of the number of photons. It will be assumed that \( \gamma \) represents the variance of the normal distribution function of the noise (as a percent of the real signal) for the transmission \( t_M \) at maximum depth (\( M = 20 \)); the corresponding variances \( \gamma_m \) at lower depths, are calculated by assuming that the number of photons is proportional to the transmission \( (\gamma_m = \gamma \sqrt{\frac{t_M}{t_m}}) \). While this approximation is not accurate (since lower energy photons are absorbed quicker than higher energy ones, the number of photons is not simply proportional with \( t_m \)), it will not significantly affect the result; the M noise-affected transmission data are calculated using

\[
t'_m = t_m (1 + \gamma_m rG)
\]

(8)

where \( rG \) is a random Gaussian-distributed variable, with unity dispersion (generated here by the Box-Muller method) \([14]\). The noise due to scatter radiation was also neglected; using a collimator after the attenuation filter might reduce significantly this type of noise, which is more difficult to be accounted for.

In what follows, for the Varian 18 MV spectrum, we will assume \( E_{\text{min}} = 0 \) and \( E_{\text{max}} = 20 \) MeV, with a bin width of 0.25 MeV (as in \([12]\)). We will also use the boundary conditions \( c_1 = c_N = 0 \) (with \( c_1 \) corresponding to \( E = 0 \)) for all the spectral reconstructions. To implement a practical method for spectral reconstruction from attenuation measurements in a clinical environment, the absorption coefficients \( \mu \) \((E) \) should be monotonic functions of the energy and the phantom should not be excessively thick. The latter requirement is for the convenience only, while the former is related to the fact the method cannot distinguish between two wavelengths that have the same absorption coefficient. Typical materials used for beam hardening (e.g. Pb or Cu) have small Half Value Layer (HVL) values, but have a
maximum in absorption around 4 MeV and 7 MeV, respectively; hence they cannot be employed for reconstruction of spectra with higher maximum photon energies. Al and C have monotonically decreasing absorption coefficients, but a HVL of about 12 or 25 cm, respectively, for 18 MeV. In this article, we will consider that the narrow beam is transmitted through a water phantom, and attenuation measurements are calculated at 20 equally spaced distances. The maximum thickness of 50 cm of water (or water equivalent) is chosen only for convenience; this phantom can be prepared easily and with a sufficiently high purity. The absorption coefficients used to calculate the beam attenuation for various water thicknesses are based on the NIST Report 5632 [15], interpolated with Mathematica software written by Schweppe [16].

2.2. Pitfalls of commonly used methods for spectral reconstruction

In this article we will limit our attention to algorithms based on the numerical inversion of the system of linear equations which – at least in principle – could describe any (continuous) real spectrum with any desired accuracy, such as Truncated Singular Valued Decomposition (TSVD) [13], Expectation-Maximization (EM) [11] and Waggener’s Iterative (WI) method [10].

The EM reconstruction is very accurate, if a very good initial estimate of the spectrum is known [11]. The main advantage over WI consists in providing a smooth distribution of the final $c_n$ values. However, if no information of the initial $c_n$ values is available, both WI and EM reconstruction algorithms fail to reproduce the spectrum accurately. (The EM solution is, however, much more accurate.) In Fig. 1, the spectral reconstructions are performed assuming constant initial values for $c_n$ ($c_n^i = 2.5$ for $n = 2$ to $N - 1$), and the boundary conditions, $c_1 = c_N = 0$. For the EM algorithm, about 40 000 iterations have
been performed, until a minimum has been obtained for the differences between the real and calculated transmitted data [11]. This number of iterations is much larger than the one (about 500) employed by Sidky et al. [11], which indicates that the starting “guess spectrum” used previously [11] was much closer to the exact solution than the flat spectrum used here to initialize the iterations. The SVD solution (which does not use initial values for $c_n$) is extremely accurate; however, it will be shown later that it is strongly affected by even very low levels of noise in the transmission data.

The major pitfall of the EM and WI algorithms occurs, therefore, only when an initial guess of the spectrum is not available (in this case, however, the former is much more accurate than the later), while SVD is not accurate in the presence of noise. The purpose of this article is to propose a novel method, which is able to accurately reconstruct X-ray spectra from transmission measurements affected by noise, when no initial “guess spectra” are available. The method is based on a search for a smooth function, which minimizes a suitably defined penalty function, and performs reasonably well, even when the transmission data are affected by noise.

2.3. The novel algorithm for spectral reconstruction based on Total Variation Regularization (TVR)

The main reason for the failure of both iterative methods (EM and WI) is that an underdetermined system admits an infinity of solutions (the matrix operator $A$ defined by Eq. (5a) has a nonempty null space); therefore, the final solution depends on the initial guess of the spectrum and on the algorithm used. In WI method, the adjustment of the coefficients has been done iteratively in such a manner that a minimum of the function:

$$D(c) = \frac{1}{M} \sum_{m=1}^{M} \sqrt{\left(\frac{t_{m,\text{calculated}} - t_{m,\text{measured}}}{t_{m,\text{measured}}}\right)^2}$$  \hspace{1cm} (9)$$

is obtained, where the $t_{m,\text{calculated}}$ represent the transmission data calculated for the spectral estimation (described by the coefficients $c_n$), while the $t_{m,\text{measured}}$ represent the transmission data calculated for the “true” spectrum. $D(c)$ represents the average relative error of estimation the input transmission data. The reason for using this relation, (instead of the more commonly-used least-squares), is to account more evenly for all the transmission data (the intensity of the beam at high depths might be much smaller than at the surface); note that $t_{m,\text{measured}}$ is always positive.

Because the Megavoltage spectrum is a smooth function, instead of searching for one of the many $c_n$ distributions that minimizes Eq. (9), we will look for a function that is smooth and provides sufficiently small differences between $t_{m,\text{calculated}}$ and $t_{m,\text{measured}}$. To implement this numerically, we will define a new contribution to the penalty function, which increases for non-smooth functions. A standard regularization procedure used in the inverse problem theory is to add to the penalty function a term related to the local gradient of the function (the Sobolev $H^1$ penalty functional) [13]:

$$D_R(c) = \frac{1}{M} \sum_{m=1}^{M} \sqrt{\left(\frac{t_{m,\text{calculated}} - t_{m,\text{measured}}}{t_{m,\text{measured}}}\right)^2} + \alpha \frac{1}{N} \sum_{n=2}^{N} (c_n - c_{n-1})^2$$  \hspace{1cm} (10)$$

where $\alpha$ is an unknown regularization parameter, which indicates how important the smoothness of the spectrum $c(E)$ is for the minimization. In this approach, the minimization of the penalty function $D_R(c)$ will not lead to a solution of the system of Eqs (3), but to an approximate solution which is, however, smooth. On the other hand, whereas an infinity of solutions exist for the system (3), which is underdetermined, the minimization of Eq. (10) admits one and only one solution ($D_R(c)$ is convex); that
solution can be easily found numerically [13]. Common algorithms employed to find numerical solution for the Total Variation Regularization of the inverse problem include the Steepest Descent Method, the Newton’s Method or the Lagged Diffusivity Fixed Point Iteration [13]. The results reported in this article have been obtained by using a simple and intuitive Monte-Carlo minimization algorithm, in which successive random adjustments of the coefficients $c_n$ are tested and only the ones that minimize $D_R(c)$ are retained. We will adjust $c_n$ in reverse order, starting with $c_N$ and ending with $c_1$; whenever a negative $c_n$ value is obtained, it will be set to zero. As it will be observed later, this method provides in many cases a good estimate for the maximum photon energy. Also, when an update of $c_n$ was successful, the same update was tried for $c_{n-1}$; this procedure took advantage of the smoothness of the spectrum and increased the speed of the algorithm. This simple method provides a good solution and is very fast (a few seconds on a regular PC). Of course, since the solution of Eq. (10) is unique, any other convex minimization algorithm is suitable for finding the solution.

Clearly $\alpha = 0$ will lead to any solution that minimizes Eq. (10), not necessarily smooth, while $\alpha \to \infty$ will lead to a flat spectrum. In Fig. 2 are plotted some reconstructed spectra, employing various values for $\alpha$; the reconstruction is not accurate for either very small or very large values of $\alpha$.

Therefore, the most important task is to find a suitable value for the regularization parameter $\alpha$ for an arbitrary (unknown) spectral distribution. For each value of $\alpha$, a set of $c'_n$ that minimize Eq. (10) was obtained, and in Fig. 3a the corresponding values of $D(c')$ (Eq. (9)) and of the roughness of the reconstructed spectrum ($\sim \frac{1}{N} \sum_{n=2}^{N} (c_n - c_{n-1})^2$) are plotted, as functions of $\alpha$. As expected, for small $\alpha$ values (of the order of $1 \times 10^{-4}$ in Fig. 3a), a rough solution was obtained, while for large $\alpha$ values (of the order of $2 \times 10^{-3}$ in Fig. 3a) a smooth function was obtained, but $t_{c,\text{calculated}}$ does not approximate...
Fig. 3. (a) Penalty function ($D(c)$, Eq. (9)) and curve roughness for the reconstructed spectra, as functions of the regularization parameter $\alpha$. (b) The Penalty function ($D_R(c)$, Eq. (10)) and the Figure of Merit of the reconstructed spectra, as functions of the regularization parameter $\alpha$. (c) Spectral reconstruction using various values of the regularization parameter, not very far from the value $\alpha = 4 \times 10^{-4}$ provided by the minimization of $D_R(c)$.

The best compromise seems to be for intermediate $\alpha$ values; in this region, the penalty function $D_R(c', \alpha)$ (calculated for the optimal set of coefficients $c'$, obtained via minimization) has a local minimum (see Fig. 2b). The value of $\alpha$, that minimizes the penalty function, seems also to produce a very good spectral estimation, as shown by the dependence on $\alpha$ of the Figure of Merit FOM:

$$FOM(\alpha) = \frac{\sqrt{\sum_n (c'_n(\alpha) - c'_{n0})^2}}{\sqrt{\sum_n (c'_{n0})^2}}$$ (11)
where $c_n'(\alpha)$ is the set of coefficients that minimizes Eq. (10) and $c_n^0$ is the “true spectrum” (see Fig. 2b). It should be noted that, in general, the true spectrum is not available (hence the FOM cannot be calculated), but a good estimate for $\alpha$ can be find from the minimization of $D_R (c', \alpha)$. When the transmission data are affected by noise, the minimization of $D_R (c', \alpha)$ lead to different $\alpha$ values ($\alpha$ increases with noise levels). These values have been used for all the spectral reconstructions presented in what follows, in spite of the fact that, in general, they did not minimize the FOM.

Note that an accurate determination of the optimal $\alpha$ value is not crucial; in Fig. 3c the reconstructed spectra are compared with the original for various values of $\alpha$. While the minimization procedure described above ($\alpha = 4.0 \times 10^{-4}$) provided the best fit, reasonable agreements have been obtained for $\alpha = 2.0 \times 10^{-4}$ and $\alpha = 8.0 \times 10^{-4}$. The FOM of the latter two spectral estimations is about 30–50 % larger than that corresponding to the best value chosen for $\alpha$; this is not much if one takes into account that the FOM for the EM reconstruction is about one order of magnitude larger. As a matter of fact, even the spectra from Fig. 2 (for which $\alpha$ has been either under or overestimated by two orders of magnitude), have a better FOM than the spectral reconstruction provided by EM.

3. Results

3.1. Spectral reconstruction using attenuation data affected by noise

There is no practical need to develop reconstruction algorithms if perfectly accurate attenuation measurements would be available. As a matter of fact, the Moore – Penrose pseudoinverse (SVD)
provides, in this case, an excellent solution to the inverse problem, as it was shown in Fig. 1. Furthermore, if exact data were available, one could ask for $M = N$ measurements, which would make the system of equations determined and hence exactly solvable. Note that, because $\mu(E)$ is a slowly varying function of $E$, the $M$ equations would almost be identical, and the determinant would almost vanish. Consequently, even slight errors in $t_m$ for noise-affected data will be amplified drastically in the system inversion.

As discussed, the SVD algorithm works very well for noise-free data. For data affected by noise, direct SVD reconstruction is extremely inaccurate (by many orders of magnitude), but there are several regularization schemes that improve its performance. In Fig. 4 the performance of the Truncated Singular Value Decomposition (TSVD) [13] has been employed for $\gamma = 10^{-6}$. This method involved the selection of the cut-off value and the setting to zero of the diagonal elements of the pseudoinverse matrix that were smaller than this cut-off. Since we know the “true” spectrum, we selected the cut-off value that provided the best Figure of Merit Eq. (11); the optimal result (plotted in Fig. 4) has been obtained by employing only the largest 6 out of the 20 diagonal values. Since it is very unlikely to have experimental attenuation data with an accuracy better than $\gamma = 10^{-6}$, the TSVD method is in general not very accurate for spectral reconstruction.

The situation is different for the TVR algorithm. In this case, minimization of the penalty function, Eq. (10), will not provide a solution of the system (3), but a smooth function that is almost a solution. To examine how the methods work, in Fig. 5a–d we plotted the reconstructed spectra for typical levels of noise, $\gamma = 0.1\%$, $\gamma = 0.5\%$, $\gamma = 1.0\%$ and $\gamma = 5\%$. The corresponding EM and TSVD solutions are also presented; for EM the number of iterations that minimize $D(c)$ (Eq. (8)) have been used [11], while for TSVD the largest 4 (for $\gamma = 0.1\%$, $\gamma = 0.5\%$) and 3 (for $\gamma = 1.0\%$ and $\gamma = 5\%$) diagonal values have been used.

In all the cases investigated, the TVR solution is more accurate than both EM and TSVD solutions. While the latter two solutions do not recover the maximum energy of the photons, the TVR solution is able to obtain a good estimate of it, at least when the attenuation data are sufficiently accurate.

3.2. Evaluation of the reconstruction algorithms (TVR, TSVD and EM) for calculating transmission in fat, muscle and fat tissue

All the algorithms for the numerical inversion of the linear system discussed here will provide calculated transmissions in excellent agreement with input transmissions (the average error being of the order of 0.005%). Therefore, all the spectra reconstructed via EM, TSVD or TVR will provide very good calculations for the transmissions through water, whether or not the estimated spectra resemble the true spectrum.

A more useful test for reconstruction algorithms consists in the comparison of the attenuation of the estimated spectra to that of the “true” spectrum through 50 cm of bone, muscle or fat tissue. The “true” spectrum is, as above, based on the Monte-Carlo calculations, using the BEAM code, of Sheikh-Bagheri and Rogers [12], for a Varian linear accelerator and 18 MV. We assumed that the input transmission data are affected by noise, with $\gamma = 0.1\%$. The calculations are performed (for bone, muscle and fat tissue) by using the spectrum reconstructed via the present method, the spectrum reconstructed by the EM and TSVD methods, and also by a very simple approximation, which is sometimes used for quick dose estimations [17]. The last approximation is employed only as an estimate of an upper bound in the errors due to a total lack of knowledge of spectral distribution (apart from the acceleration potential). This approximation, considered a “thumb rule” for quick dose calculation, approximates the X-ray beam
Fig. 5. Spectral reconstruction using the TVR (thick line), TSVD (squares) and EM (triangles) algorithm, for computer-generated transmission data, with added random Gaussian noise: (a) $\gamma = 0.1\%$, (b) $\gamma = 0.5\%$, (c) $\gamma = 1.0\%$ and (d) $\gamma = 5\%$. The true spectrum obtained via Monte Carlo calculations in [12] for Varian 18 MV is represented by circles.

with a flux of monoenergetic photons, with an energy of one third of that of the maximum photon energy. The monoenergetic beam that best fit the attenuation measurements at the surface (for our spectrum) would have the energy 5.7 $MeV$, while the one that best fit the attenuation measurements at a depth of 25 cm of water would have the energy 6.6 $MeV$; hence using a value of 6 $MeV$ is justifiable (of course, approximating a polyenergetic spectrum by a monoenergetic one is never accurate). The intensity of the “monoenergetic” approximation has been obtained by fitting the first transmission data.

The relative errors for dose calculations, plotted in Fig. 6, are in all cases calculated with respect to the transmissions of the “true” spectral distribution provided in [12]. While the “rule of thumb” generates
errors up to about 8% (open triangles, stars and circles, Fig. 6), the TSVD (filled triangles, stars and circles) and EM (half-filled triangles, stars and circles) methods reduces these errors to about 1%. The relative errors of the TVR solutions (white triangles, stars and circles) presented in the inset of Fig. 6 (at a different scale), are less than about 0.2%.

3.3. Spectral reconstruction for other spectra

All the spectra calculated via Monte-Carlo in the article of Sheikh-Bagheri and Rogers [12] have been reconstructed using the EM, TSVD and TVR algorithms, using $M = 20$ transmissions data at equidistant depths in water; the maximum water depth is again 50 cm. It was assumed that $\gamma = 0.1\%$. The maximum energies used in calculations were 5 MeV for Varian 4 MV spectrum, 8 MeV for Varian and Elekta 6 MV spectra, 9 MeV for Siemens 6 MV spectrum, 12 MeV for Varian 10 MV spectrum, 17 MeV for Varian 15 MV spectrum, and 20 MeV for Siemens 18 MV and Elekta 25 MV spectra. For the acceleration potentials of 6 MV or less, a better TVR solution has been obtained by imposing also $c_2 = 0$. In all the case investigated, the TVR algorithm provided a better FOM than both TSVD and EM. In most of the cases, TVR provided also a good estimate of the maximum energy of the photons, unlike the other two algorithms.

4. Summary and conclusions

It is relatively straightforward to reconstruct a megavoltage X-ray spectrum, when very accurate transmission data are known (using the SVD solution). Such data are, however, not available in practice,
Fig. 7. Spectra calculated via Monte-Carlo for various manufacturers and acceleration potentials (reported in [12]) (circles) are used to test the algorithms for spectral reconstruction; a random Gaussian noise with $\gamma = 0.1\%$ was assumed for all the computer-generated transmission data. Spectral reconstructions using the TVR (thick line), TSVD (squares) and EM (triangles) algorithms were obtained for: (7a) Varian 4 MV; (7b) Varian 6 MV; (7c) Elekta 6 MV; (7d) Siemens 6 MV; (7e) Varian 10 MV; (7f) Varian 15 MV; (7g) Siemens 18 MV; (7h) Elekta 25 MV.

and in this case the direct inversion methods typically fail. For noise-affected data, the previously proposed iterative algorithms (the WI and EM methods) are much more accurate than the direct inversion methods, providing that very good initial guesses of the spectra are available. However, in some cases, these accurate guesses are not available, either.

In this article, we presented a novel method for estimating megavoltage X-ray spectra from attenuation measurements, without any prior knowledge of the spectral distribution (apart from the fact that all the coefficients are positive and that the spectrum is a smooth function). The method is based on a search
for a minimum of a penalty function, which has a term proportional to the distance to the transmission data, and a term proportional to the “roughness” of the reconstructed spectrum. Minimization of this penalty leads to a smooth function, which approximates the real spectrum well. It is also shown that the spectrum reconstructed using the present method, using noise-affected transmission data and without using any initial guess spectrum, can be successfully used to predict adsorption through bone, muscle and fat tissue. The relative errors (compared to the absorption of the real spectrum) obtained are in general a few times smaller than the ones obtained from TSVD or EM algorithms. The algorithm has been applied to other Megavoltage spectra (4 MV, 6 MV, 10 MV, 15 MV and 25 MV), for transmission data affected by $\gamma = 0.1\%$ noise and outperformed EM and TSVD in all the cases investigated. The robustness of the algorithm, which provides reasonable accurate reconstructions even for transmission data affected by 5% noise, makes it suitable for spectral reconstruction in a typical clinical environment.
Acknowledgements

This work was supported by the National Institute of Health Grant # P20-CA126201.

References
